

# A Note On The Spectral Norms of The Circulant Matrices Connected Integer Number Sequences

Durmuş Bozkurt\*

Department of Mathematics, Science Faculty of Selçuk University

January 20, 2013

## Abstract

In this paper, we compute the spectral norms of the matrices related with integer sequences and we give some example related with Fibonacci, Lucas, Pell and Perrin numbers.

Keywords: Fibonacci number, Lucas numbers, Pell numbers, Perrin numbers, integer number sequence, spectral norm.

MS Classification Number: 15A60, 15F35, 15B36, 15B57

## 1 Introduction

In [1], the upper and lower bounds for the spectral norms of the matrices with Fibonacci and Lucas numbers are obtained by S. Solak. In [2], Ipek obtained as  $\|A\|_2 = F_{n+1} - 1$  and  $\|B\|_2 = F_{n+2} + F_n - 1$  the spectral norms of the matrices by Solak defined in [1] where  $F_i$  is the  $i$ th Fibonacci number.

Let  $A$  be any  $n \times n$  complex matrix. The well known the spectral norm of the matrix  $A$  is

$$\|A\|_2 = \sqrt{\max_{1 \leq i \leq n} |\lambda_i(A^H A)|}$$

where  $\lambda_i(A^H A)$  is eigenvalue of  $A^H A$  and  $A^H$  is conjugate transpose of the matrix  $A$ .

By a circulant matrix of order  $n$  is meant a square matrix of the form

$$C = \text{circ}(c_0, c_1, \dots, c_{n-1}) = \begin{bmatrix} c_0 & c_1 & c_2 & \dots & c_{n-1} \\ c_{n-1} & c_0 & c_1 & \dots & c_{n-2} \\ c_{n-2} & c_{n-1} & c_0 & \dots & c_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_1 & c_2 & c_3 & \dots & c_0 \end{bmatrix}. [3]$$

---

\*e-mail: dbozkurt@selcuk.edu.tr

Now we define our matrix.  $(x_n)$  is any positive integer numbers sequence and  $x_i$  is  $i$ th the component of the sequence  $(x_n)$  for  $i = 0, 1, 2, \dots$ . Let matrix  $A_x$  be following form:

$$A_x = \text{circ}(x_0, x_1, \dots, x_{n-1}). \quad (1)$$

The main objective of this paper is to obtain the spectral norms of the matrices  $A_x$  in (1).

## 2 Main Result

**Theorem 1** *Let the matrix  $A_x$  be as in (1). Then*

$$\|A_x\|_2 = \sum_{i=0}^{n-1} x_i.$$

where  $\|\cdot\|_2$  is the spectral norm and  $x_j$  are  $j$ th components of the sequence  $(x_n)$ .

**Proof.** Since the circulant matrices are normal, the spectral norm of the circulant  $A_x$  is equal to its spectral radius. Also the circulant  $A_x$  is irreducible and its entries are nonnegative. Therefore the spectral radius of the matrix  $A_x$  is the same its Perron root.

Let  $y$  be a vector with all the components 1. Then

$$A_x y = \left( \sum_{i=0}^{n-1} x_i \right) y.$$

Since  $\sum_{i=0}^{n-1} x_i$  is an eigenvalues of  $A_x$  corresponding a positive eigenvector, it must be the Perron root of the matrix  $A_x$ . Then we have

$$\|A_x\|_2 = \sum_{i=0}^{n-1} x_i.$$

The proofs are completed. ■

## 3 Numerical Consideration

The well known  $F_n, L_n, P_n$  and  $R_n$  are  $n$ th Fibonacci, Lucas, Pell and Perrin numbers with recurrence relations

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_n = F_{n-1} + F_{n-2} & \text{if } n > 1 \end{cases}, \quad L_n = \begin{cases} 2 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ L_n = L_{n-1} + L_{n-2} & \text{if } n > 1 \end{cases}$$

$$P_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ P_n = 2P_{n-1} + P_{n-2} & \text{if } n > 1 \end{cases}, R_n = \begin{cases} 3 & \text{if } n = 0 \\ 0 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ R_n = R_{n-2} + R_{n-3} & \text{if } n > 2 \end{cases},$$

respectively.

If the sequence  $(x_n)$  is the Fibonacci and Lucas sequences then the following results are obtained:

$$\|A_F\|_2 = \sum_{i=0}^{n-1} F_i = F_{n+1} - 1 \quad [2]$$

and

$$\|A_L\|_2 = \sum_{i=0}^{n-1} L_i = F_{n+2} + F_n - 1 \quad [2]$$

If the sequence  $(x_n)$  is Pell and Perrin sequences then we have

$$\|A_P\|_2 = \sum_{i=0}^{n-1} P_i = \frac{1}{2}(P_n + P_{n-1} - 1)$$

and

$$\|A_R\|_2 = \sum_{i=0}^{n-1} R_i = R_{n+4} - 1.$$

## References

- [1] S.Solak, On the norms of circulant matrices with the Fibonacci and Lucas numbers, Appl. Math. and Comput., 160 (2005), 125-132.
- [2] A. İpek, On the spectral norms of circulant matrices with classical Fibonacci and Lucas numbers entries, Appl. Math. Comput. 217 (2011) 6011–6012.
- [3] P. J. Davis, Circulant Matrices, A Wiley-Interscience Publication, 1979.